Answers

NAME: ______________________

1. Solve the following initial value problems.

(a) $\frac{dN}{dt} = \frac{t+2}{t}$ for $t \geq 1$ with $N(1) = 2$.

$$\frac{dN}{dt} = 1 + \frac{2}{t} \Rightarrow N(t) = t + 2\ln(t) + C$$

$$2 = N(1) = 1 + 2\ln(1) + C$$

$$2 = 1 + C$$

$$C = 1 \Longrightarrow \boxed{N(t) = t + 2\ln(t) + 1}$$

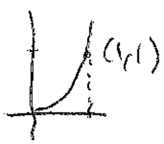
(b) $\frac{dy}{dx} = \frac{e^{-x} + e^{x}}{2}$ for $x \geq 0$ with $y(0) = 0$.

$$y(x) = \frac{-e^{-x}}{2} + \frac{e^x}{2} + C \qquad \boxed{y(x) = \frac{e^x - e^{-x}}{2}}$$

$$0 = y(0) = \frac{e^0 - e^{-0}}{2} + C$$

$$= 0 + C \Rightarrow C = 0$$

2. Approximate the area under the parabola $y = x^2$ from 0 to 1 using four equal subintervals.



$$\int_0^1 x^2 dx \approx \frac{1}{4}\left[f(0) + f\left(\tfrac{1}{4}\right) + f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{4}\right)\right]$$

(From Left)

$$= \frac{1}{4}\left(0 + \frac{1}{16} + \frac{1}{4} + \frac{9}{16}\right)$$

$$= \frac{1}{4}\left(\frac{5}{8} + \frac{1}{4}\right)$$

$$= \frac{1}{4}\left(\frac{7}{8}\right) = \boxed{\frac{7}{32}}$$

3. Use Leibniz's Rule to solve $y = \frac{d}{dx}\int_x^{2x} (1 + t^2)dt$.

$$\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt = f(g(x))g'(x) - f(h(x))h'(x)$$

$$\Longrightarrow y = (1 + (2x)^2)(2) - (1 + x^2)(1)$$

$$= 2 + 8x^2 - 1 - x^2 = \boxed{7x^2 + 1}$$

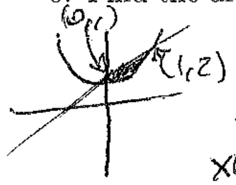
4. Find the average value of $f(x) = -x^2 - 2x + 5$ over the interval $[-4, 0]$.

$$\frac{1}{0-(-4)}\int_{-4}^{0} -x^2 - 2x + 5 \, dx$$

$$= \frac{1}{4}\left(\frac{-x^3}{3} - x^2 + 5x \Big|_{-4}^{0}\right)$$

$$= \frac{1}{4}\left(0 - \left(\frac{64}{3} - 16 - 20\right)\right) = \frac{1}{4}\left(36 - \frac{64}{3}\right) = 9 - \frac{16}{3} = \boxed{\frac{11}{3}}$$

5. Find the area of the region bounded by $y = x^2 + 1$ and $y = x + 1$.



$x^2+1 = x+1$
$x^2 = x$
$x^2 - x = 0$
$x(x-1) = 0$
$x = 0, 1$

$$\int_0^1 \left((x+1)-(x^2+1)\right)dx = \int_0^1 -x^2+x \, dx$$
$$= \left[-\frac{x^3}{3} + \frac{x^2}{2}\Big|_0^1\right] = \left[\left(-\frac{1}{3}+\frac{1}{2}\right) - \left(\frac{0}{3}+\frac{0}{2}\right)\right]$$
$$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

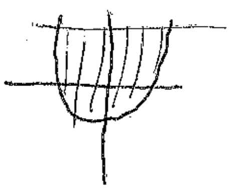
6. Find the volume of the solid bounded by $y = -x^2 + 1$, $y = 0$, and rotated about the x-axis

$-x^2+1 = 0$
$x^2 = 1$
$x = \pm 1$

Circle as cross-sectional area $\Rightarrow$ Disk Method

$$\pi \int_{-1}^{1} (1-x^2)^2 dx = \pi \int_{-1}^{1} 1 - 2x^2 + x^4 dx = \pi\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \Big|_{-1}^{1}\right)$$
$$= \pi\left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right] = \pi\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \boxed{\frac{16\pi}{15}}$$

7. Imagine stacking squares whos sides are the length between $f(x) = 2x^2 - 1$ and $g(x) = 7$. This would create a shape over the area between $f(x)$ and $g(x)$. What is the volume of this solid?



Area of a square: s^2
Cross-sectional area

$2x^2 - 1 = 7 \Rightarrow 2x^2 = 8$
$x^2 = 4 \Rightarrow x = \pm 2$

$$\int_{-2}^{2} \left((7) - (2x^2-1)\right)dx = \int_{-2}^{2} 8 - 2x^2 dx = \left[8x - \frac{2}{3}x^3 \Big|_{-2}^{2}\right]$$
$$= \left(\left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)\right) = \boxed{32 - \frac{32}{3}}$$

8. Solve the indefinite integral $\int x^3 e^{x^2} dx$.

$y = x^2 \Rightarrow dy = 2x\,dx$

$$\Rightarrow \frac{1}{2}\int y e^y dy$$

$u = y$ $\quad dv = e^y dy$
$du = dy$ $\quad v = e^y$

$$\Rightarrow = \frac{1}{2}\left(ye^y - \int e^y dy\right) = \frac{1}{2}\left(ye^y - e^y + C\right)$$

$$= \frac{e^y}{2}(y-1) + C = \boxed{\frac{e^{x^2}}{2}(x^2-1) + C}$$