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Problem 1. Use the properties of definite integrals to solve the integrals.

a) Given $\int_1^0 \ln(x)dx = \int_1^e \ln(x)dx$, find $\int_0^e \ln(x)dx$.

$$\int_0^e \ln(x)dx = \int_0^1 \ln(x)dx + \int_1^e \ln(x)dx$$
$$= -\int_1^e \ln(x)dx + \int_1^e \ln(x)dx = \boxed{0}$$

b) Given $\int_0^a x^3dx = \frac{a^4}{4}$ find $\int_1^2 2x^3dx$.

$$\int_0^2 x^3dx = \int_0^1 x^3dx + \int_1^2 x^3dx$$
$$\Rightarrow \frac{2^4}{4} = \frac{1^4}{4} + \int_1^2 x^3dx \Rightarrow \frac{16-1}{4} = \frac{15}{4} = \int_1^2 x^3dx$$
$$\Rightarrow \int_1^2 2x^3dx = 2\left(\frac{15}{4}\right) = \boxed{\frac{15}{2}}$$

c) Given $\int_0^a x^4dx = \frac{a^5}{5}$, find $\int_{-1}^1 \frac{x^4}{2}dx$.

$$\int_{-1}^1 x^4 = -\int_0^{-1} x^4 + \int_0^1 x^4 = -\left(\frac{-1}{5}\right) + \left(\frac{1}{5}\right) = \frac{2}{5}$$
$$\text{so } \int_{-1}^1 \frac{x^2}{2} = \frac{1}{2}\left(\frac{2}{5}\right) = \boxed{\frac{1}{5}}$$

d) Given $\int_0^a \sin(x)dx = 1 - \cos(a)$ and that $f(x) = \sin(x)$ is an odd function (this means that $\sin(-x) = -\sin(x)$), find $\int_1^\pi \sin(-x)dx$.

$$\int_0^\pi \sin(x)dx = \int_0^1 \sin(x)dx + \int_1^\pi \sin(x)dx$$
$$1 - \cos(\pi) = (1 - \cos(1)) + \int_1^\pi \sin(x)dx$$
$$= 2$$
$$\Rightarrow 1 + \cos(1) = \int_1^\pi \sin(x)dx$$
$$\text{so } \int_1^\pi \sin(-x)dx = -\int_1^\pi \sin(x)dx = \boxed{-1 - \cos(1)}$$

$$\frac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\,h'(x) - f(g(x))\,g'(x)\right]$$

Problem 2. Use Leibniz's rule to solve the following integrals.

a) $\frac{d}{dx}\int_1^x (1+t)dt$ $\quad (1+x)(1)-(1+1)(0)$

$= 1+x$

b) $\frac{d}{dx}\int_x^3 (1+t)dt$ $\quad (1+3)(0)-(1+x)(1)$

$= -1-x$

c) $\frac{d}{dx}\int_{2-x^2}^{x+x^3} (t^2-1)dt$ $\quad = 3x^8+7x^6+2x^5+5x^4-8x^3-2x^2+6x-1$

$-\int_0^{2-x^2}(t^2-1)dt + \int_0^{x+x^3}(t^2-1)dt$

$= -((2-x^2)^2-1)(-2x) + ((x+x^3)^2-1)(1+3x^2)$

$= (2x)(3-4x^2+x^4) + (x^2+2x^4+x^6-1)(1+3x^2)$

$= 2x^5-8x^3+6x+3x^8+7x^6+5x^4-2x^2-1$

d) $\frac{d}{dx}\int_{x^3-2x}^{1+x^2} (t+1)dt$

$= (1+x^2+1)(2x)-(x^3-2x+1)(3x^2-2)$

$= 2x^3+4x-(3x^5-8x^3+3x^2+4x-2)$

$= -3x^5+10x^3-3x^2+2$