Answers To Week 1's Worksheet

October 1, 2019

Problem 1. Find the general solutions for the following differential equations

a) $\frac{dy}{dx} = \frac{3}{x^2} - 2x^2$

Rewrite the equation without the fractions by turning the exponent negative

$$= 3x^{-2} - 2x^2$$

Now find the antiderivative by reversing the power rule

$$y(x) = -3x^{-1} - \frac{2x^3}{3} + C = \frac{-3}{x} - \frac{2x^3}{3} + C$$

The negative in front of the 3 comes from the -1 exponent and the fraction 1/3 in the second term comes from the 3 exponent.
Check your answer by finding the derivative of your answer.

b) $\frac{dy}{dx} = \frac{1+2x}{x}$

Since we only have 1 term on the bottom, we can split up the fraction

$$= \frac{1}{x} + \frac{2x}{x} = \frac{1}{x} + 2$$

Find the antiderivative by reversing the power rule and reversing $\frac{d}{dx}\ln(x) = \frac{1}{x}$

$$y(x) = \ln|x| + 2x + C$$

Check your answer by finding the derivative of your answer.

c) $\dfrac{dy}{dx} = \dfrac{1}{1+2x}$

Find the antiderivative by reversing $\dfrac{d}{dx}\ln(x) = \frac{1}{x}$

$$y(x) = \frac{\ln|1+2x|}{2} + C$$

Dividing by 2 comes from the extra 2 leftover after applying the chain rule when finding the derivative of $\ln|1+2x|$.
Check your answer by finding the derivative of your answer.

d) $\dfrac{dy}{dx} = sec^2(3x-1)$

We know that $\dfrac{d}{dx}\tan(x) = \sec^2(x)$. Therefore we will find the antiderivative by reversing this.

$$y(x) = \frac{\tan(3x-1)}{3} + C$$

Dividing by 3 comes from the extra 3 leftover after applying the chain rule when finding the derivative of $\tan(3x-1)$.
Check your answer by finding the derivative of your answer.

e) $\dfrac{dy}{dx} = \dfrac{1}{x\ln(2x)}$ [Hint: consider $\dfrac{dy}{dx}\ln(\ln(x))$.]

Let's check our hint by finding the derivative

$$\frac{dy}{dx}\ln(\ln(x)) = \frac{1}{\ln(x)} * \frac{1}{x}$$

The first term is the derivative of the outside log and the second term is the derivative of the inside log (chain rule). Hence we get

$$\frac{1}{x\ln(x)}$$

which is super close to what we want. Since the only difference is $2x$ instead of x, we know we will at least need $\ln(2x)$ on the inside of our answer. So a good guess would be $\ln(\ln(2x))$. We should check our answer to see if we need to divide or multiply by any extra constants like in previous problems.

$$\frac{d}{dx}\ln\ln(2x) = \frac{1}{\ln(2x)} * \frac{2}{2x} = \frac{1}{\ln(2x)} * \frac{1}{x} = \frac{1}{x\ln(2x)}$$

which is exactly what we want so we do not need to multiply or divide by any extra constants. Hence our answer is

$$y(x) = \ln\ln(2x) + C$$

Problem 2. Solve the initial value problem that models the growth of a population with time:

$$\frac{dN}{dt} = t$$

where $N(0) = 20$.

To solve the initial value problem we first need to find the general solution to the differential equation (what we did in the previous problems). We can do this by reversing the power rule.

$$N(t) = \frac{t^2}{2} + C$$

Dividing by 2 comes from the extra 2 we get when bring down the exponent.
Now that we have this we can plug in our initial value and solve for C.

$$20 = N(0) = \frac{(0)^2}{2} + C = \frac{0}{2} + C = C$$

Therefore $C = 20$. So our solution to the initial value problem is

$$N(t) = \frac{t^2}{2} + 20$$

Problem 3. Solve the initial value problem

$$\frac{dy}{dx} = -2x^2 + 3$$

where $y(3) = 10$.

We will reverse the power rule again to find the following

$$y(x) = \frac{-2x^3}{3} + 3x + C$$

Then plug in the initial value

$$10 = y(3) = \frac{-2(3)^3}{3} + 3(3) + C = \frac{-2(27)}{3} + 9 + C = -18 + 9 + C$$

$$\Rightarrow 10 + 18 - 9 = C \Rightarrow 19 = C$$

Hence

$$y(x) = \frac{-2x^3}{3} + 3x + 19$$

Problem 4. An object moves with a constant acceleration of 12 m/s^2. Find the how far the object has fallen after 2 seconds if the object has the initial velocity $v(0) = 4\ m/s$ and initial displacement of $d(0) = 2\ m$.

Recall that the derivative of displacement ($d'(t) = v(t)$) is velocity and the derivative of velocity is acceleration ($v'(t) = a(t)$). So we have $a(t) = 12\frac{m}{s^2}$ and taking the antiderivative we get

$$v(t) = 12t + C$$

Using our initial value we can find C

$$4 = v(0) = 12(0) + C = C \Rightarrow v(t) = 12t + 4$$

Then find the antiderivative again by reversing the power rule

$$d(t) = 6t^2 + 4t + C$$

Using the initial value we can find C

$$2 = d(0) = 6(0)^2 + 4(0) + C = C \Rightarrow d(t) = 6t^2 + 4t + 2$$

Now that we have our solution we can plug in our time and find our answer

$$d(2) = 6(2)^2 + 4(2) + 2 = 6(4) + 8 + 2 = 24m$$