Warm up

Find (x, y) that satisfies

$$x - y = 5$$

and

$$2x + y = 1$$

Systems of Equations

Suppose you have multiple equations.

$$x - y = 5$$

$$2x + y = 1$$

Is there an x and a y that satisfies both?

2 ways to solve this:

Substitution

Isolate a variable & substitute into the other equation

$x = y + 5$

$2(y+5) + y = 1$

$3y + 10 + y = 1$

$3y = -9$

$y = -3$

Plug into either equation

$x - (-3) = 5$

$x + 3 = 5$

$x = 2$

Elimination (Addition)

Add equations together to cancel a variable

$$\begin{array}{r} x - y = 5 \\ 2x + y = 1 \\ \hline 3x + 0 = 6 \end{array}$$

$x = 2$

Plug into either equation

$(2) - y = 5$

$-y = 3$

$y = -3$

$(2, -3)$ is a solution

Ex (Recommend to use elimination)

$4x + 3y = 11$

$-5x + 2y = 15$

Elimination: Need coefficients to (almost) match ← off by a negative

$5(4x + 3y) = 5(11)$

$4(-5x + 2y) = 4(15)$

$20x + 15y = 55$

$-20x + 8y = 60$

$23y = 115$

$y = 5$

Plug it back in →

$4x + 3(5) = 11$

$4x + 15 = 11$

$4x = -4$

$x = -1$

$(-1, 5)$ is a solution

2-Variable Word Problems

Concert Ticket Prices. One evening 1500 concert tickets were sold for the Fairmont Summer Jazz Festival. Tickets cost $25 for a covered pavilion seat and $15 for a lawn seat. Total receipts were $28,500. How many of each type of ticket were sold?

What 2 things is it asking for? Let those be x & y.

2 types of tickets: x = pavilion seats

y = lawn seats

$x + y = 1500$ # of tickets

$25x + 15y = 28,500$ money/value of sold tickets

$-15(x + y) = -15(1500)$

$-15x - 15y = -22,500$

$25x + 15y = 28,500$

$10x + 0 = 6,000$

$x = 600$

$(600) + y = 1500$

$y = 900$

600 covered Pavillion seat tickets sold

900 lawn seat tickets sold

Motion. A DC10 airplane travels 3000 km with a tailwind in 3 hr. It travels 3000 km with a headwind in 4 hr. Find the speed of the plane and the speed of the wind.

Assuming headwind & tailwind are the same wind.

2 things it's asking for?

x = Speed of airplane

y = Speed of wind

$$3x + 3y = 3000$$
$$4x - 4y = 3000$$

$$x + y = 1000$$
$$x - y = 750$$

$$2x = 1750$$
$$x = 875$$

Sub: $(875) - y = 750$

$$875 - 750 = y$$
$$125 = y$$

$$x + y = 1000$$
$$x - y = 750 \rightarrow x = 750 + y$$

Sub: $(750 + y) + y = 1000$

$$750 + 2y = 1000$$
$$2y = 250$$
$$y = 125$$

Sub: $x - (125) = 750$

$$x = 875$$

Nutrition. One serving of tomato soup contains 100 Cal and 18 g of carbohydrates. One slice of whole wheat bread contains 70 Cal and 13 g of carbohydrates. (*Source*: U.S. Department of Agriculture) How many servings of each would be required to obtain 230 Cal and 42 g of carbohydrates?

2 things its asking for?

x = Servings of tomato soup

y = Slices of whole wheat bread

$$100x + 70y = 230$$
$$18x + 13y = 42$$

Simplify a bit

$$10x + 7y = 23$$
$$18x + 13y = 42$$

$$18(10x + 7y) = 18(23)$$
$$-10(18x + 13y) = -10(42)$$

$$180x + 126y = 414$$
$$-180x - 130y = -420$$
$$-4y = -6$$
$$y = \frac{-6}{-4} = \frac{3}{2}$$

$10x + 7y = 23 \rightarrow 10x = 23 - 7y$

$$x = \frac{23 - 7y}{10}$$

Sub: $18\left(\frac{23 - 7y}{10}\right) + 13y = 42$

$\times 10$: $18(23 - 7y) + 130y = 420$

$$414 - 126y + 130y = 420$$
$$4y = 420 - 414$$
$$4y = 6$$
$$y = \frac{6}{4} = \frac{3}{2}$$

$\Rightarrow 10x + 7\left(\frac{3}{2}\right) = 23$

$\times 2$: $20x + 7(3) = 46$

$$20x = 46 - 21 = 25$$
$$x = \frac{25}{20} = \frac{5}{4}$$

Sub: $10x + 7\left(\frac{3}{2}\right) = 23$

$$\vdots$$

$$x = \frac{5}{4}$$