Warm up

Solve $x^4 = 1$.

± 1 yes but that's only 2 roots. What happened to the other roots?

$x^4 - 1 = 0$ $\qquad a^2 - b^2 = (a+b)(a-b)$

$= (x^2-1)(x^2+1)$

$= (x-1)(x+1)(x^2+1) = 0$

What are the roots of $x^2+1=0$?

Complex Numbers

$i = \sqrt{-1}$ is called an imaginary number.

$$i = \sqrt{-1}$$

$$i^2 = (\sqrt{-1})^2 = (\sqrt{-1})(\sqrt{-1}) = -1$$

$$i^3 = i^2 \cdot i = (-1)(i) = -i$$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

Ex

- $i^5 = i^4 \cdot i = 1 \cdot i = i$
- $i^{10} = i^{4+4+2} = i^4 \cdot i^4 \cdot i^2 = (1)(1)(-1) = -1$
- $i^{29} = i^{28+1} = i^{28} i = (i^4)^7 \cdot i = (1)^7 i = 1 \cdot i = i$
- $i^{63} = i^{60+3} = i^{60} i^3 = (i^4)^{15} i^3 = (1)^{15} i^3 = 1 \cdot i^3 = -i$
- $i^{100} = (i^4)^{25} = 1^{25} = 1$

Defn

A complex number is a number of the form $a+bi$ where a and b are real numbers.

$$7-3i$$

$7=a$ is called the real part

$-3=b$ is called the imaginary part

Addition & Subtraction aren't too bad, however, multiplication & division are a bit more complicated.

Goal: write it in the form $a+bi$

Addition & Subtraction

Basically just add/subtract the "real parts" together & add/subtract the "imaginary parts"

Ex

① $(8+6i)+(3+2i)$

$=(8+3)+(6+2)i$

$=11+8i$

② $(4+5i)-(6-3i)$

$=(4-6)+(5-(-3))i$

$=-2+8i$

Before we multiply complex numbers first consider an easier example (no real part)

Ex: $\sqrt{-16}\cdot\sqrt{-25}=\sqrt{16}\sqrt{-1}\sqrt{25}\sqrt{-1}=(4i)(5i)=20(i^2)=-20$

Recall: $(a+b)(c+d) \overset{\text{foil}}{=} ac + ad + bc + bd$

$$(7+3)(6-5) = (7)(6) + (7)(-5) + (3)(6) + (3)(-5) \quad \leftarrow \text{Key part foil}$$
$$= 42 - 35 + 18 - 15$$
$$= 7 + 3$$
$$= 10$$

We need to foil when multiplying complex numbers.

Ex

① $(3-5i)(8-2i)$

$$= (3)(8) + (3)(-2i) + (-5i)(8) + (-5i)(-2i)$$
$$= 24 - 6i - 40i + 10i^2$$
$$= 24 - 6i - 40i - 10$$
$$= 14 - 46i$$

② $(1+2i)(1+3i)$

$$= 1 + 3i + 2i + (2i)(3i)$$
$$= 1 + 3i + 2i + 6i^2$$
$$= 1 + 5i - 6$$
$$= -5 + 5i$$

Dividing Complex Numbers

Recall: $(a+b)(a-b) = a^2 - b^2$

So $(3+2i)(3-2i) = 9 - 4i^2 = 9 + 4 = 13$ gets rid of i

Defn

The complex conjugate of $a+bi$ is $a-bi$.

Multiplying a complex number by its complex conjugate will get rid of your imaginary part.

$$(a+bi)(a-bi) = a^2 - b^2 i^2 = a^2 + b^2 \quad \text{no more } i$$

Ex

① $\dfrac{i}{2+i}\cdot\dfrac{2-i}{2-i} = \dfrac{i(2-i)}{(2+i)(2-i)} = \dfrac{2i-i^2}{4-i^2} = \dfrac{2i+1}{4+1} = \dfrac{1+2i}{5} = \boxed{\dfrac{1}{5}-\dfrac{2}{5}i}$

② $\dfrac{4+i}{-3-2i}\cdot\dfrac{-3+2i}{-3+2i} = \dfrac{(4+i)(-3+2i)}{(-3-2i)(-3+2i)} = \dfrac{-12+8i-3i+2i^2}{9-4i^2} = \dfrac{-12+5i-2}{9+4}$

$= \dfrac{-14+5i}{13} = \boxed{\dfrac{-14}{13}+\dfrac{5}{13}i}$

③ $\dfrac{8-i}{1-2i}\cdot\dfrac{1+2i}{1+2i} = \dfrac{(8-i)(1+2i)}{1^2-(2i)^2} = \dfrac{8+16i-i-2i^2}{1-4i^2} = \dfrac{8+15i+2}{1+4}$

$= \dfrac{10+15i}{5} = \dfrac{10}{5}+\dfrac{15}{5}i = \boxed{2+3i}$