Interval Notation

Pick a number between 7 & 32.

Can safely choose 8, 9, 10, ⋯, 31, 9.5, 12.346

Did I include 7? Did I include 32?

Brackets: [,] – include

Parentheses: (,) – don't include

$[7, 32]$ includes 7 & 32 & everything in between

$(7, 32)$ does not include 7 & does not include 32
But does contain everything in between
Such as 7.00001 & 31.99999

Can mix notation

$[7, 32)$ includes 7 & does not include 32

$(7, 32]$ does not include 7 & include 32

In general:

	Names
• $[a,b]$ = all x such that $a \leq x \leq b$	"closed"
• $[a,b)$ = all x such that $a \leq x < b$	"half-open/closed"
• $(a,b]$ = all x such that $a < x \leq b$	"half-open/closed"
• (a,b) = all x such that $a < x < b$	"open"

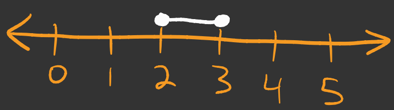
Notice $a < b$. $[8, -2)$ does not make sense. It must be of the form $(-2, 8]$ as $-2 < 8$.

More Notation

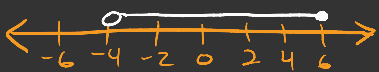
$\{x \mid a \le x \le b\}$
$\{x : a \le x \le b\}$ are read "x such that $a \le x \le b$"

Ex

- $\{x \mid 2 \le x \le 3\} = [2, 3]$

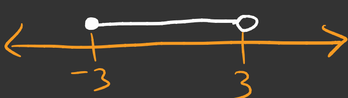


- $\{x \mid -4 < x \le 6\} = (-4, 6]$



- $\{x \mid 7 < x < 30.3\} = (7, 30.3)$

- $\{x \mid -3 \le x < 3\} = [-3, 3)$



You can do this with $\pm\infty$

Ex

$\{x \mid x \ge \pi\} = [\pi, \infty)$

doesn't really make sense to include ∞ so we use parenthesis

$\{x \mid x \le \pi\} = (-\infty, \pi)$

Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

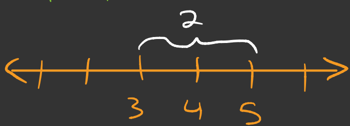
What do absolute values show us?

The <u>absolute value</u> of a number, a, is distance between a & 0.

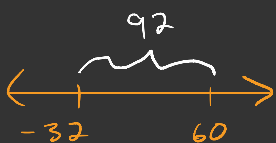
More generally, the distance between two numbers, a and b, is $|a-b|$.

Ex

- $|3-5| = |-2| = 2 \Rightarrow$ 3 & 5 have a distance of 2.



- $|-32-60| = |-92| = 92 \Rightarrow$ -32 & 60 have distance 92



Integer Exponents

For any positive integer, n, $a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ times}}$

Ex

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$6^1 = 6$

$4^0 = 1$

Important examples

1) $a^0 = 1$

2) $a^{-n} = \frac{1}{a^n}$

negative exponent: reciprocal & make exponent positive

Ex

- $3^{-6} = \frac{1}{3^6}$
- $\frac{1}{3^{-6}} = \frac{1}{\frac{1}{3^6}} = 3^6$ reciprocal of reciprocal is itself

  $1 \div \frac{1}{3^6}$

Can do some fraction manipulating:

- $\frac{5^{-4}}{3^{-2}} = \frac{3^2}{5^4}$
- $\frac{x^3 y^{-8}}{z^{-10}} = \frac{x^3 z^{10}}{y^8}$
- $\frac{a^n}{b^m} = \frac{b^{-m}}{a^{-n}}$

Properties of Exponents

- $a^m \cdot a^n = a^{m+n}$ Product Rule
  - $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$

- $\frac{a^m}{a^n} = a^{m-n}$ Quotient Rule
  - $\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$

- $(a^m)^n = a^{m \cdot n}$ Power Rule
  - $(2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$

- $(ab)^m = a^m b^m$ Power of a Product
  - $(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$

- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ Power of a Quotient
  - $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

Examples

1) $y^{-5} \cdot y^3 = y^{-5+3} = y^{-2} = \frac{1}{y^2}$

2) $\frac{48x^{12}}{16x^4} = \frac{48x^{12}x^{-4}}{16} = \frac{48x^{12-4}}{16} = \frac{48x^8}{16} = 3x^8$

3) $(2s^{-2})^5 = 2^5 s^{-2 \cdot 5} = 2^5 s^{-10} = \frac{2^5}{s^{10}}$